OCR
RECOGNISING ACHIEVEMENT

Wednesday 25 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Fig. 1 shows a particle of mass 0.5 kg hanging from a light vertical spring. At time t seconds its displacement is x m below its equilibrium level and its velocity is $v\,\mathrm{m\,s^{-1}}$ vertically downwards. The forces on the particle are

- its weight, $0.5g$ N,
- the tension in the spring, $2.5(x + 0.2g)$ N,
- the resistance to motion, kv N, where k is a positive constant.

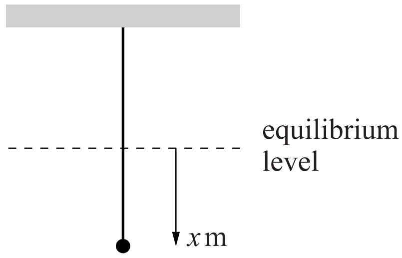

Fig. 1

(i) Use Newton's second law to write down the equation of motion for the particle, justifying the signs of the terms. Hence show that the displacement is described by the differential equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 5x = 0.$$ **[4]**

The particle is initially at rest with $x = 0.1$.

(ii) Find the set of values of k for which the system is

(A) over-damped, (B) under-damped, (C) critically damped.

In each of the cases (A) and (B), sketch a possible displacement-time graph of the motion. **[7]**

(iii) Sketch a displacement-time graph of the motion of the particle in the case $k = 0$. **[1]**

A subsequent motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \sin 4t.$$

(iv) Find the particular solution subject to the conditions that the particle is initially at rest with $x = 0$. **[12]**

2 A population of bacteria grows from an initial size of 1000. After t hours the size of the population is P. After 10 hours the size of the population is 4000.

At first the rate of growth is modelled as being proportional to the size of the population.

(i) Write down a differential equation modelling the population growth and solve it for P in terms of t. **[4]**

To allow for constraints on the population growth, the model is revised to give

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP(5000 - P),$$

where k is a constant.

(ii) Solve this differential equation to find t in terms of P, subject to the given conditions. **[9]**

(iii) Find the time it takes for the population to reach 4900, giving your answer in hours, correct to two decimal places. **[1]**

The model is further refined to give

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 10^{-15}P^{\alpha}(5000 - P),$$

where α is a constant, and it is observed that the maximum *rate* of growth occurs when $P = 4000$.

(iv) Show that $\alpha = 4$. **[5]**

Starting from $t = 10$, $P = 4000$, Euler's method is used with a step length of 0.2 to solve this differential equation. The algorithm is given by $t_{r+1} = t_r + h$, $P_{r+1} = P_r + h\dot{P}_r$.

(v) Continue the algorithm for two steps to estimate the size of the population when $t = 10.4$. **[5]**

3 Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y = x.$$

(i) Sketch the isoclines $\frac{\mathrm{d}y}{\mathrm{d}x} = m$ for $m = 0, \pm1, \pm2$. Hence draw a sketch of the tangent field. **[3]**

(ii) State which of the isoclines is an asymptote to any solution curve. **[1]**

(iii) Sketch on your tangent field the solution curves through (2, 0) and (0, –2). **[3]**

(iv) Use the integrating factor method to solve the differential equation for y in terms of x, subject to the condition $y = 3$ when $x = 0$. **[7]**

Now consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y = \sin x.$$

(v) Find the complementary function and a particular integral. Hence state the general solution. **[6]**

(vi) Find the solution subject to the condition $y = 3$ when $x = 0$ and sketch the solution curve. **[4]**

4 The simultaneous differential equations

$$\frac{dx}{dt} = -x + 2y$$

$$\frac{dy}{dt} = -x - 4y + e^{-2t}$$

are to be solved.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[14]**

(ii) Find the corresponding general solution for y. **[3]**

Initially $x = 5$ and $y = 0$.

(iii) Find the particular solutions. **[4]**

(iv) Show that $\frac{y}{x} \rightarrow -\frac{1}{2}$ as $t \rightarrow \infty$. Show also that there is no value of t for which $\frac{y}{x} = -\frac{1}{2}$. **[3]**

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Wednesday 25 January 2012 – Afternoon

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4758/01 Differential Equations

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1(i)	
1(ii)(*A*)	
1(ii)(*B*)	

1(ii)(*C*)	
1(iii)	
1(iv)	
	(answer space continued overleaf)

1(iv)	(continued)

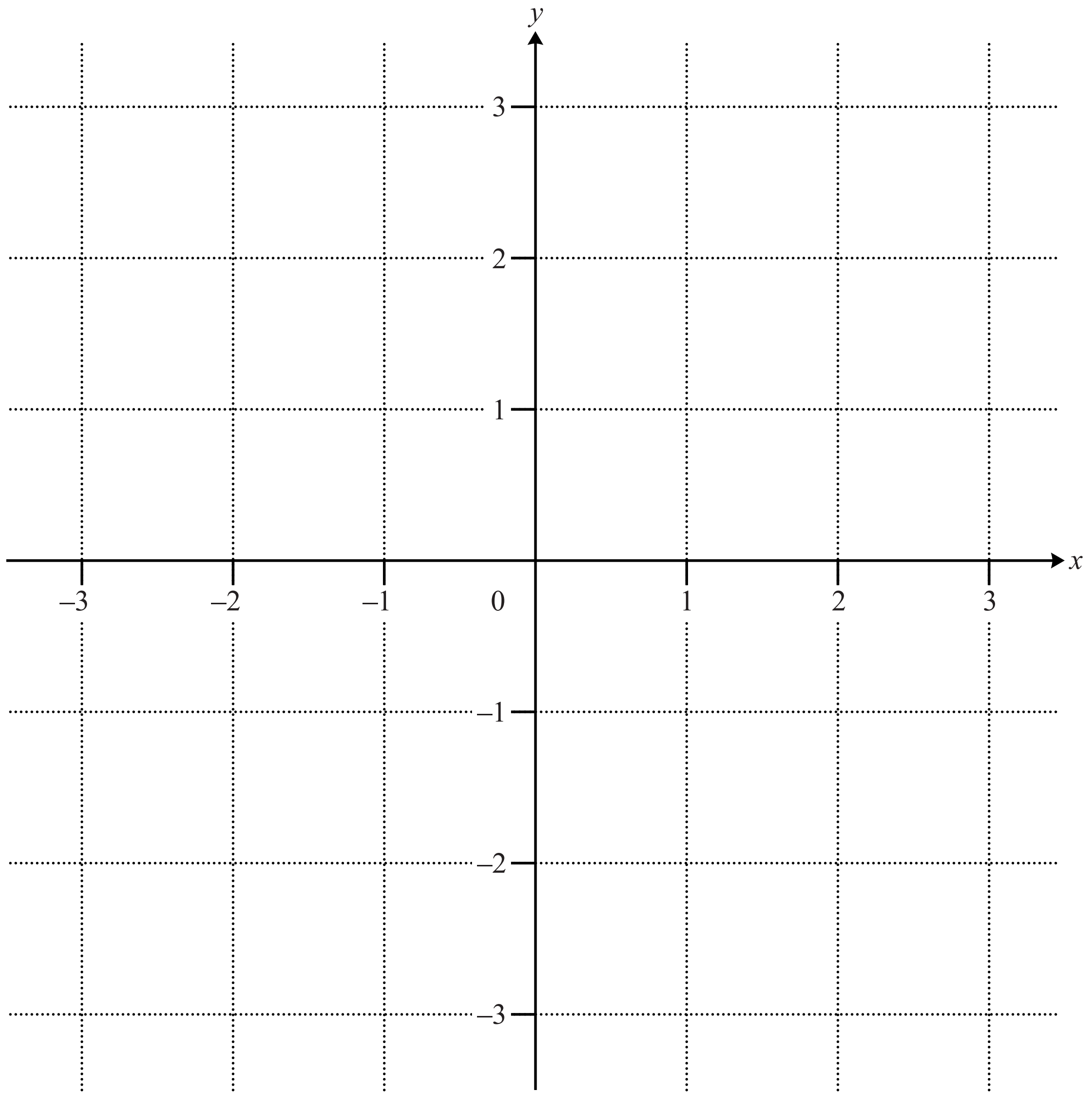
2(i)

2(ii)

2(iii)	
2(iv)	

2(v)	

3(i) & (iii)



A spare copy of this diagram can be found on page 16.

3(ii)

3(iv)	

3(v)	
3(vi)	

4(i)	

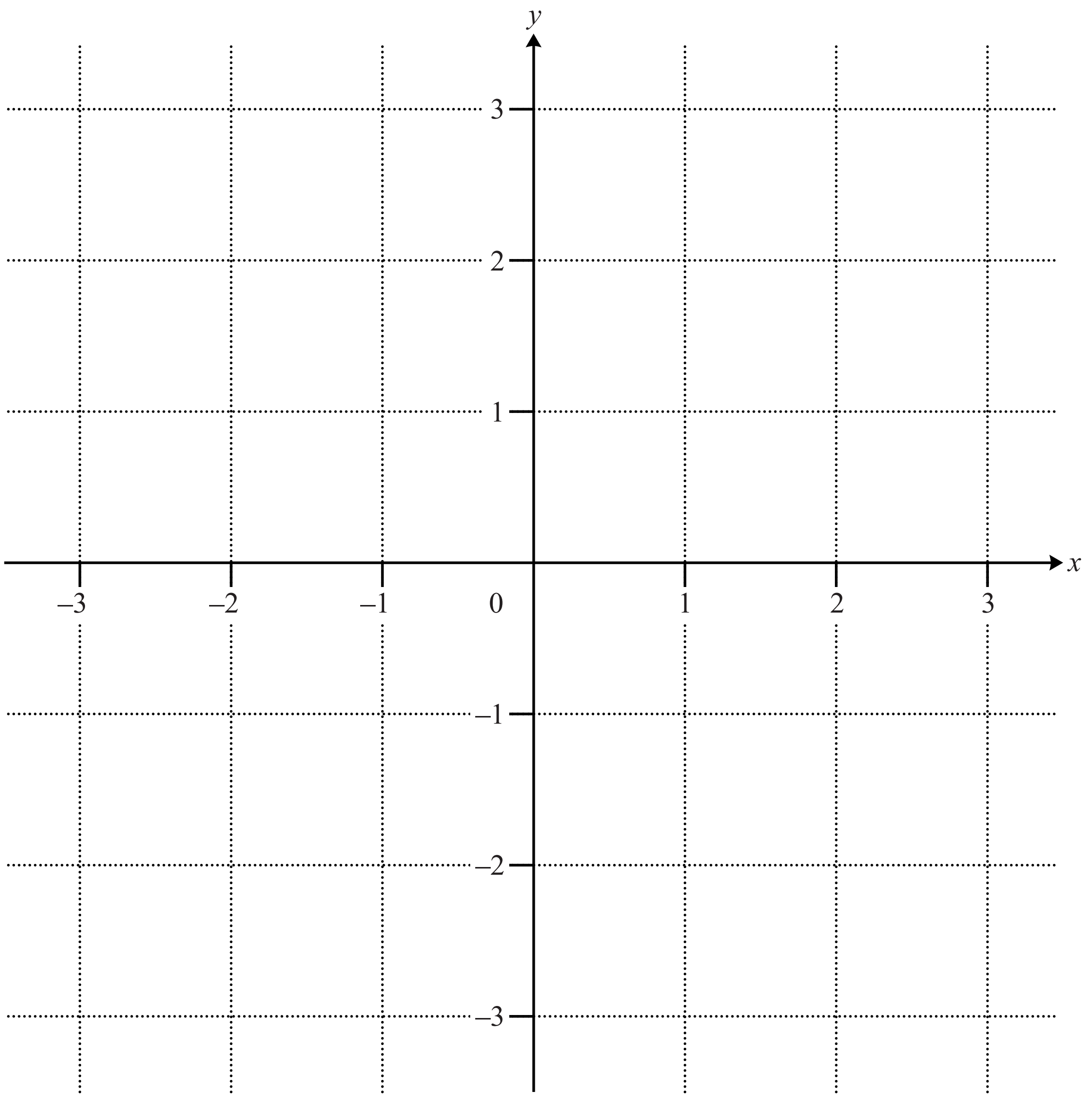
4(i)	(continued)
	(answer space continued overleaf)

4(i)	(continued)
4(ii)	
4(iii)	

4(iii)	**(continued)**
4(iv)	
	(answer space continued overleaf)

4(iv)	(continued)

Spare copy of diagram for questions 3(i) and 3(iii)



RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4758:** Differential Equations

Mark Scheme for January 2012

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

12. **Subject-specific Marking Instructions**

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (eg lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

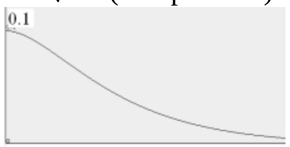
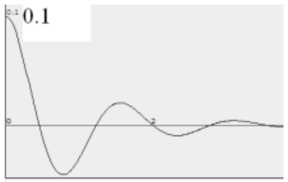
Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

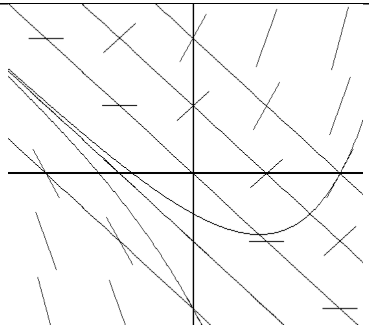
i. If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

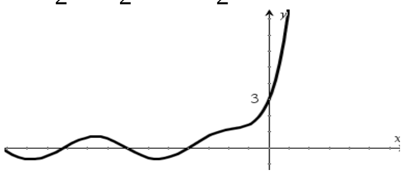
j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$ma = 0.5g - kv - 2.5(x + 0.2g)$	M1 A1	N2L at least one force term present
			(at least) weight down, tension up, resistance opposes motion	B1	All 3 forces mentioned
			$0.5\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -k\frac{\mathrm{d}x}{\mathrm{d}t} - 2.5x$		
			$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 0$	E1	May be awarded without the B1 earned
				[4]	
1	(ii)	(*A*)	$(2k)^2 - 4\times5 > 0$	B1	Or equivalent in *(B)* or *(C)*
			$k > \sqrt{5}$ (as k positive)	B1	Ignore $-\sqrt{5}$
				B1	Shape
		(*B*)	$(0 <) k < \sqrt{5}$	B1	FT incorrect k in *(A)* Ignore $-\sqrt{5}$
				B1	Shape
				B1	Initial conditions. May be awarded in either (A) or (B)
		(*C*)	$k = \sqrt{5}$	B1	FT
				[7]	
1	(iii)			B1	Horizontal intercepts not required. At least one complete oscillation. 0.1 marked
				[1]	

Question			Answer	Marks	Guidance
1	(iv)		$\lambda^2+2\lambda+5=0$	M1	
			$\lambda=-1\pm 2j$	A1	
			CF $\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)$	F1	
			PI $x=a\cos 4t+b\sin 4t$	B1	
			in DE: $-16ac-16bs-8as+8bc+5ac+5bs=s$	M1	Differentiate and substitute
				M1	Compare coefficients and solve
			$a=-\frac{8}{185},\ b=-\frac{11}{185}$	A1	
			$x=-\frac{1}{185}\left(8\cos 4t+11\sin 4t\right)+\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)$	F1	PI + CF with two arbitrary constants
			$\dot{x}=-\frac{1}{185}\left(-32\sin 4t+44\cos 4t\right)$ $+\mathrm{e}^{-t}\left(-A\cos 2t-B\sin 2t-2A\sin 2t+2B\cos 2t\right)$	M1	Differentiate (product rule)
			$t=0, x=0 \Rightarrow 0=-\frac{8}{185}+A$	M1	Use condition
			$t=0, \dot{x}=0 \Rightarrow 0=-\frac{44}{185}-A+2B$	M1	Use condition
			$A=\frac{8}{185},\ B=\frac{26}{185}$		
			$x=\frac{1}{185}\left(-8\cos 4t-11\sin 4t+\mathrm{e}^{-t}\left(8\cos 2t+26\sin 2t\right)\right)$	A1	cao
				[12]	
2	(i)		$\frac{\mathrm{d}P}{\mathrm{d}t}=kP$		
			$P=A\mathrm{e}^{kt}$	B1	
			initial size $A=1000$	M1	Use condition
			$4000=1000\,\mathrm{e}^{10k}$	M1	Use condition
			$P=1000\,\mathrm{e}^{(0.1\ln 4)t}$	A1	Or $1000\times 4^{0.1t}$ Accept numerical equivalent of $0.1\ln 4$ Award for correct value of k seen
				[4]	

Question			Answer	Marks	Guidance
2	(ii)		$\int \frac{1}{P(5000-P)}\,\mathrm{d}P = \int k\,\mathrm{d}t$	M1	Separate
			$\frac{1}{5000}\int\left(\frac{1}{P}+\frac{1}{5000-P}\right)\mathrm{d}P = kt+c$	M1*	Partial fractions
			$\frac{1}{5000}\left(\ln P - \ln(5000-P)\right) = kt+c$	M1 dep*	Integrate
				A1	LHS
				B1	RHS (need constant on one side)
			$t=0, P=1000 \Rightarrow \frac{1}{5000}\ln\frac{1}{4} = c$	M1	Use condition
			$t=10, P=4000 \Rightarrow \frac{1}{5000}\ln 4 = 10k+c$	M1	Use condition
			$k = \frac{1}{50000}\ln 16$	A1	
			$t = \frac{10}{\ln 16}\ln\left(\frac{4P}{5000-P}\right)$	A1	aef
				[9]	
2	(iii)		$P = 4900 \Rightarrow t = \frac{10}{\ln 16}\ln\left(\frac{4\times 4900}{100}\right) \approx 19.04$	B1	FT on partial fractions used in part (ii)
				[1]	
2	(iv)		Growth rate max when $P^{\alpha}(5000-P)$ max	M1	
			$\frac{\mathrm{d}}{\mathrm{d}P}\left(5000P^{\alpha} - P^{\alpha+1}\right) = 0$	M1*	Differentiate
			$\Rightarrow 5000\alpha P^{\alpha-1} - (\alpha+1)P^{\alpha} = 0$	A1	
			so for $P>0$, $5000\alpha = (\alpha+1)P$		
			occurs when $P = 4000$		
			so $5000\alpha = 4000(\alpha+1)$	M1 dep*	
			$\Rightarrow \alpha = 4$	E1	
				[5]	

Question			Answer	Marks	Guidance
2	(v)		t = 10, P = 4000, P' = 256, hP' = 51.2 t = 10.2, P = 4051.2, P' = 255.57, hP' = 51.114 t = 10.4, P = 4102.3 Population ≈ 4102	M1 A1 A1 A1 A1 **[5]**	
3	(i)			M1 M1 A1 **[3]**	Isoclines Direction indicators Allow small inaccuracies
3	(ii)		$x+y=-1$	B1 **[1]**	aef Award for m =-1 seen
3	(iii)		See (i)	M1 A1 A1 **[3]**	Attempt either curve Curve through (2, 0) Curve through (0, –2)
3	(iv)		$I=\mathrm{e}^{\int -1\,\mathrm{d}x}=\mathrm{e}^{-x}$ $\mathrm{e}^{-x}\frac{\mathrm{d}y}{\mathrm{d}x}-\mathrm{e}^{-x}y=x\mathrm{e}^{-x}$ $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-x}y\right)=x\mathrm{e}^{-x}$ $\mathrm{e}^{-x}y=-x\mathrm{e}^{-x}-\mathrm{e}^{-x}+A$ $x=0, y=3 \Rightarrow A=4$ $y=4\mathrm{e}^{x}-x-1$	B1 M1 M1 A1 M1 M1 A1 **[7]**	 Multiply by IF Integrate (parts) Including constant Use condition Divide by IF cao

Question			Answer	Marks	Guidance
3	(v)		$\lambda - 1 = 0 \Rightarrow \lambda = 1$	M1	
			CF Ae^x	A1	
			PI $y = a\cos x + b\sin x$	B1	
			$\frac{dy}{dx} = -a\sin x + b\cos x$		
			$-a\sin x + b\cos x - (a\cos x + b\sin x) = \sin x$	M1	Differentiate, substitute and compare
			$a = b = -\frac{1}{2}$	A1	
			GS $y = Ae^x - \frac{1}{2}\cos x - \frac{1}{2}\sin x$	F1	PI + CF with one arbitrary constant
				[6]	
3	(vi)		$x = 0, y = 3 \Rightarrow 3 = A - \frac{1}{2}$	M1	Use condition
			$y = \frac{7}{2}e^x - \frac{1}{2}\cos x - \frac{1}{2}\sin x$	A1	
					
				B1	Oscillates for $x < 0$
				B1	Through (0, 3) and exp. growth for $x > 0$
				[4]	

Question			Answer	Marks	Guidance
4	(i)		$y=\frac{1}{2}(\dot{x}+x)$	M1	Express y as subject
			$\dot{y}=\frac{1}{2}(\ddot{x}+\dot{x})$	M1	Express $\dot{y}$ as subject
			$\frac{1}{2}(\ddot{x}+\dot{x})=-x-\frac{4}{2}(\dot{x}+x)+e^{-2t}$	M1	Substitute for y
				M1	Substitute for $\dot{y}$
			$\ddot{x}+5\dot{x}+6x=2e^{-2t}$	A1	
			AE $\lambda^2+5\lambda+6=0$	M1	
			$\lambda=-2$ or -3	A1	FT their AE
			CF $Ae^{-2t}+Be^{-3t}$	F1	
			PI $x=ate^{-2t}$	B1	cao
			$\dot{x}=-2ate^{-2t}+ae^{-2t}$, $\ddot{x}=4ate^{-2t}-4ae^{-2t}$	M1	Differentiate twice
			$4ate^{-2t}-4ae^{-2t}+5\left(-2ate^{-2t}+ae^{-2t}\right)+6ate^{-2t}=2e^{-2t}$	M1	Substitute
			$-4a+5a=2$	M1	Compare
			$a=2$	A1	
			GS $x=2te^{-2t}+Ae^{-2t}+Be^{-3t}$	F1	PI + CF with 2 arbitrary constants
				[14]	
4	(ii)		$\frac{dx}{dt}=-4te^{-2t}+2e^{-2t}-2Ae^{-2t}-3Be^{-3t}$	M1	
			$y=\frac{1}{2}(\dot{x}+x)$	M1	Substitute for $x, \dot{x}$
			$y=(1-t)e^{-2t}-\frac{1}{2}Ae^{-2t}-Be^{-3t}$	A1	cao
				[3]	
4	(iii)		$t=0, x=5 \Rightarrow 5=A+B$	M1	Use condition
			$t=0, y=0 \Rightarrow 0=1-\frac{1}{2}A-B$	M1	Use condition
			A = 8, B = -3		
			$x=2te^{-2t}+8e^{-2t}-3e^{-3t}$	A1	cao
			$y=-te^{-2t}-3e^{-2t}+3e^{-3t}$	A1	cao
				[4]	

Question			Answer	Marks	Guidance
4	(iv)		$\frac{y}{x}=\frac{-t-3+3\mathrm{e}^{-t}}{2t+8-3\mathrm{e}^{-t}}\to -\frac{1}{2}$	E1	Convincingly shown
			$\frac{y}{x}=-\frac{1}{2}\Leftrightarrow -t-3+3\mathrm{e}^{-t}=-t-4+\frac{3}{2}\mathrm{e}^{-t}$		
			$\Leftrightarrow \frac{3}{2}\mathrm{e}^{-t}=-1$	M1	Or attempt other valid argument e.g. $y\downarrow, x\uparrow$ so $y/x\downarrow$
			but $\mathrm{e}^{-t}>0$ for any t, so no values of t	E1 **[3]**	Complete argument

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

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Facsimile: 01223 552553

4758 Differential Equations (Written Examination)

General Comments

Candidates showed a good level of understanding of the methods of solving differential equations being examined in this paper. As always, the vast majority were able to solve second order linear differential equations and do so with a pleasing degree of accuracy. Unusually, this year candidates seemed to find it more difficult to decide on which three questions to attempt and many answered at least some parts of all four questions, presumably leaving it to the examiner to determine which three were their best. The first two parts of Question 1 and the middle parts of Question 2 proved to be stumbling-blocks for a significant number of candidates. These covered topics which are less routine and depended on an ability to apply syllabus and subject knowledge in less familiar scenarios.

Comments on Individual Questions

1 The majority of candidates seemed less than confident with the aspects of this topic that were being tested in the first three parts of the question. There was evidence that they had some relevant knowledge, but not always an ability to apply it appropriately to answer the given requests. The routine request in part (iv) was, as always, a reliable source of marks.

1 (i) There were a pleasing number of well-presented solutions to this first request, although many candidates gave little, if any, justification for the signs of the terms.

1 (ii) Responses to this part of the question were variable in quality. Candidates appeared to have some knowledge of what was required, but there was a lot of confusion when matching an appropriate set of values of k with the different damping situations. The sketches were rarely fully correct, with only a minority of candidates using the initial condition as the starting-point for their graph.

1 (iii) Very few correct sketches were seen. The common error was not to show the initial conditions.

1 (iv) This routine application of the method for the solution of a second order linear differential equation was familiar territory for the candidates and the majority earned most of the marks. The coefficients of the trigonometric terms in the particular integral, though not simple fractions, were often found correctly, displaying a pleasing accuracy in the algebra involved.

2 Attempts at this question were variable in quality, with a significant number of candidates successfully negotiating part (i), only to come to a grinding halt in part (ii) and omit the remaining parts.

2 (i) The majority of candidates who attempted this part offered a concise and accurate solution.

2 (ii) Only a minority of candidates realised that, having separated the variables, the use of partial fractions was necessary in order to integrate the integral involving P. Those who did follow this approach invariably earned most of the marks available, although often losing the final mark by not expressing t in terms of P. Those who did not use partial fractions offered a wide variety of incorrect integration methods, usually involving incorrect algebra.

2 (iii) This was omitted by the significant minority of candidates who had abandoned their attempts at part (ii)

2 (iv) Only a small number of candidates realised the need to differentiate the given expression for the derivative of *P*, in order to find the maximum *rate* of growth. The need to differentiate was highlighted by the use of italics in the question, but this strong hint was not taken.

2 (v) Candidates showed themselves to be very competent at using Euler's method. Many, however, did not round their answer to a whole number, as required by the request for a population size.

3 This question was a popular choice, with most candidates having adequate knowledge and understanding of most of the topics covered to score some marks in each part.

3 (i) Most candidates did not attempt to sketch the requested isoclines. The direction indicators were usually correct, although sometimes lacking in sufficient quantity, and often only in the right hand quadrants. A more comprehensive set of indicators would have helped in drawing the solution curves requested in part (iii).

3 (ii) This did not seem to be well-understood.

3 (iii) For those who had drawn adequate tangent fields in part (i), the task of sketching the two curves was a simple proposition and some very neat curves were seen. For others, who had worked only in the first and fourth quadrants in part (i), their solution curves were only partially correct.

3 (iv) The integrating factor method was used well by most candidates. It was particularly pleasing to see that the need for integration by parts was not a stumbling-block, the only common errors being with signs.

3 (v) Candidates had no trouble in finding the complementary function and made only numerical errors, if any, when finding a particular integral.

3 (vi) Many candidates obtained the correct solution after applying the given condition. To sketch the solution curve, candidates needed to realise that for large positive values of *x* the exponential term was dominant, and for large negative values of *x* the trigonometric terms were dominant. Some recognised one or other of these, but few offered sketches that displayed both.

4 This question was attempted by all candidates and many scored high marks.

4 (i) Candidates are very familiar with this type of question and they work through the various stages methodically. Unusually, the second order differential equation for *x* in terms of *t* was not given in the question (as a check), but it was pleasing to see that the majority worked accurately and found the correct general solution for *x*. A small number of candidates used the wrong form of particular integral.

4 (ii) Candidates knew what to do here, and the majority correctly used the product rule in differentiating their general solution for *x*.

4 (iii) The initial conditions were always applied, the main loss of marks being due to earlier slips in signs.

4 (iv) A degree of rigour was expected in an acceptable solution to this final part. Very few candidates were able to offer convincing arguments to either of the requests.